

## CUMC / CCÉM 2005

### Prime Numbers and Irreducible Polynomials

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#### Abstract / résumé

The study of prime numbers goes back to antiquity. For example, the Greeks asked if there are infinitely many primes  $p$  such that  $p + 2$  is also prime (the twin prime problem). After more than 2000 years, a substantial breakthrough was achieved recently by Goldston, Pintz, and Yıldırım on the twin prime problem. We will give a brief description of how this was done.

We will also indicate a few of the techniques developed over the centuries to study prime numbers. Some of these methods have been motivated by an analogy between integers and polynomials.

There is a strong analogy between the world of prime numbers and the world of irreducible polynomials. For example, every natural number can be written as the product of prime numbers uniquely and every polynomial with integer coefficients can be written as the product of irreducible polynomials uniquely. This analogy has been largely responsible for the parallel development of number theory and algebraic geometry in the 20th century and continues to be a source of new ideas today. In the latter part of the talk, we will explore this theme and indicate how it has helped us in understanding one of the celebrated unsolved problems of our time, namely, the Riemann hypothesis. We will also discuss how this is related to the study of gaps between prime numbers.

#### Biography / biographie

M. Ram Murty holds the Queen's Research Chair in Mathematics at Queen's University. He obtained his PhD from MIT in 1980 under the direction of Harold Stark. After that he was a member of the Institute for Advanced Study in Princeton (1980-81) and the Tata Institute for Fundamental Research in Mumbai, India (1981-82). In 1982, he joined McGill University as an NSERC University Research Fellow. He was at McGill until 1996 at which time he moved to Queen's. In 1988, he was awarded the Coxeter-James Prize by the Canadian Mathematical Society for his work on Kolyvagin's hypothesis. He was elected Fellow of the Royal Society of Canada in 1990. In 1991, NSERC awarded him the Steacie Fellowship for his work in number theory. Along with his brother, Kumar Murty, he was awarded the 1996 Balaguer Prize for his book on  $L$ -functions. In 1998, he received the Killam Prize for his work on sieve theory. In 2003, the Canadian Mathematical Society awarded him the Jeffery-Williams Prize for his general contributions to mathematics. During his research career, Professor Murty has supervised more than 60 graduate students and post-doctoral fellows.